DIGITAL SIGNAL PROCESSING

UNIT-V
FIR Filter Design

1. Characteristics of FIR Filters with Linear Phase
2. Frequency Response of Linear Phase FIR Filters
3. Window Functions and its Characteristics
   - Rectangular Window
   - Triangular (Bartlett) Window
   - Hanning (Hann) Window
   - Hamming Window
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4. Steps to Design Digital LPF/HPF/BPF/BSF through FIR Methods
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The filters designed by considering only finite samples of impulse response are called Finite Impulse Response (FIR) Filters. Design of LPF/HPF/BPF/BSF through FIR method does not involve the design of analog filter that means here directly we can design the required digital filter through Fourier series method or by using Window Techniques.

Characteristics of FIR Filters with Linear Phase:

Characteristics of a distortion less filter are

- Magnitude of frequency response or Magnitude response of a desired filter is constant.
- Phase of frequency response or Phase response of desired filter is linear.

If the frequency response of a desired filter is $H_d(e^{j\omega})$, then the characteristics of a filter are

- $H_d(e^{j\omega}) = ke^{-j\omega\alpha}$
- $|H_d(e^{j\omega})| = k = \text{Constant}$
- $\angle H_d(e^{j\omega}) = -\omega\alpha = \text{Linear}$

Linear phase FIR filters can be achieved by $h(n) = h(N-1-n)$ with $\alpha = (N-1)/2$, where $h(n)$ is the impulse response of FIR filter and N shows the number of samples of $h(n)$.

Proof:

We know that $H_d(e^{j\omega}) = ke^{-j\omega\alpha}$ and $h_d(n) = h(n)$, over the range 0 to N – 1.

$$\Rightarrow \sum_{n=-\infty}^{\infty} h_d(n)\, e^{-j\omega n} = ke^{-j\omega\alpha}$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n)\, e^{-j\omega n} = k\, e^{-j\omega\alpha}$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n)\,[\cos(\omega n) - j\sin(\omega n)] = k\,[\cos(\omega\alpha) - j\sin(\omega\alpha)]$$

$$\Rightarrow \sum_{n=0}^{N-1} [h(n)\cos(\omega n) - j\,h(n)\sin(\omega n)] = k\cos(\omega\alpha) - jk\sin(\omega\alpha)$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n)\cos(\omega n) - j\sum_{n=0}^{N-1} h(n)\sin(\omega n) = k\cos(\omega\alpha) - jk\sin(\omega\alpha)$$

Compare both real and imaginary parts both side

$$\sum_{n=0}^{N-1} h(n)\cos(\omega n) = k\cos(\omega\alpha) \text{ ----------(1)}$$

$$\sum_{n=0}^{N-1} h(n)\sin(\omega n) = k\sin(\omega\alpha) \text{ -------------(2)}$$

$Equations (2)/(1)$

$$\Rightarrow \frac{\sum_{n=0}^{N-1} h(n)\sin(\omega n)}{\sum_{n=0}^{N-1} h(n)\cos(\omega n)} = \frac{k\sin(\omega\alpha)}{k\cos(\omega\alpha)}$$

$$\Rightarrow \cos(\omega\alpha)\sum_{n=0}^{N-1} h(n)\sin(\omega n) = \sin(\omega\alpha)\sum_{n=0}^{N-1} h(n)\cos(\omega n)$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n)\sin(\omega n)\cos(\omega\alpha) = \sum_{n=0}^{N-1} h(n)\cos(\omega n)\sin(\omega\alpha)$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n)\,[\sin(\omega n)\cos(\omega\alpha) - \cos(\omega n)\sin(\omega\alpha)] = 0$$

$$\Rightarrow \sum_{n=0}^{N-1} h(n)\ \sin[\omega(n-\alpha)] = 0$$

$$\Rightarrow h(0)\sin[\omega(0-\alpha)] + h(1)\sin[\omega(1-\alpha)] + \ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + h(N-2)\sin[\omega(N-2-\alpha)] + h(N-1)\sin[\omega(N-1-\alpha)] = 0$$

Above sum is zero only when

$h(0)\sin[\omega(0-\alpha)] = -h(N-1)\sin[\omega(N-1-\alpha)]$,

$h(1)\sin[[\omega(1-\alpha)] = -h(N-2)\sin[\omega(N-2-\alpha)]$,

…….

…….

$h(n)\sin[[\omega(n-\alpha)] = -h(N-1-n)\sin[\omega(N-1-n-\alpha)]$

Compare magnitude and angle of sine function

$h(n) = h(N-1-n)$ ------------- It is the condition for linear phase FIR filters

and

$\sin[[\omega(n-\alpha)] = -\sin[\omega(N-1-n-\alpha)]$

$\Rightarrow \sin[[\omega(n-\alpha)] = \sin[-\omega(N-1-n-\alpha)]$

$\Rightarrow \omega(n-\alpha)] = -\omega(N-1-n-\alpha)$

$\Rightarrow (n-\alpha) = -(N-1-n-\alpha)$

$\Rightarrow n-\alpha+N-1-n-\alpha = 0$

$\Rightarrow -2\alpha+N-1 = 0$

$\Rightarrow 2\alpha = N-1$

$\Rightarrow \alpha = (N-1)/2$ --- It is the constant used in the linear phase FIR filter design

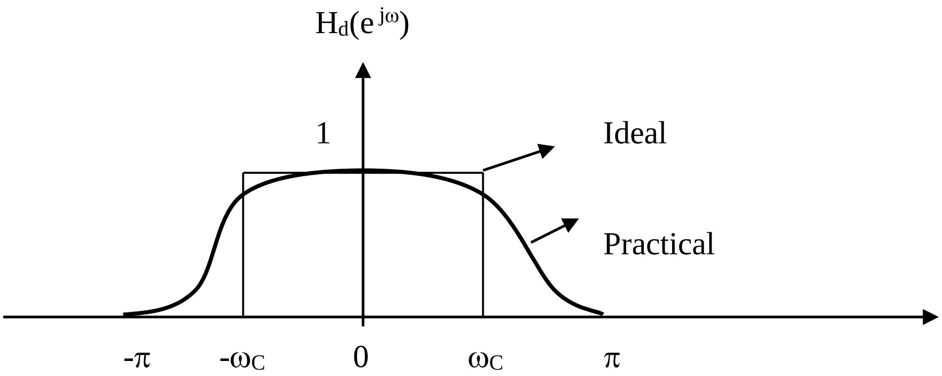
Linear phase FIR filters can be achieved by $h(n) = h(N-1-n)$ with $\alpha = (N-1)/2$

Frequency Response of Linear Phase FIR Filters:

Frequency response of LPF/HPF/BPF/BSF as shown below

(A)Low Pass Filter:

Low pass filter allows only low frequency signals over the range $-\omega_C \le \omega \le \omega_C$ and attenuate all other high frequency signals. Frequency response of ideal and practical characteristics of a low pass filter as shown below.



$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha}, & -\omega_c \le \omega \le \omega_c \\ 0, & -\pi \le \omega \le -\omega_c \text{ and } \omega_c \le \omega \le \pi \end{cases}$$

Impulse response of a desired filter can be obtained from IDTFT

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega\alpha}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega(n-\alpha)}\, d\omega$$

$$= \frac{1}{2\pi}\left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi}\frac{e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}}{j(n-\alpha)}$$

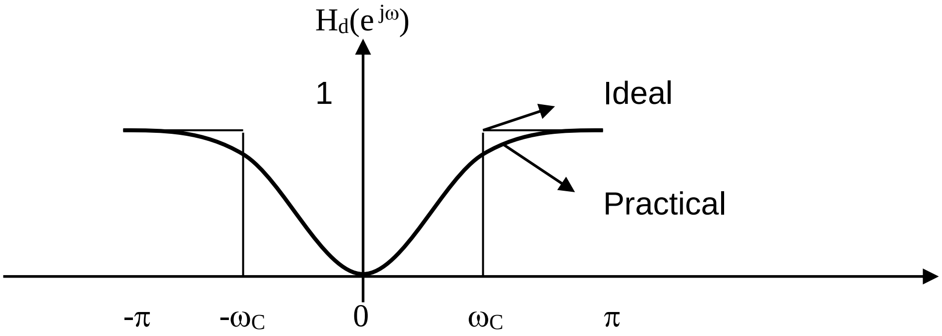
$$= \frac{1}{2\pi}\frac{2j\sin[\omega_c(n-\alpha)]}{j(n-\alpha)}$$

$$= \begin{cases} \dfrac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}, & \text{if } n \ne \alpha \\ \dfrac{\omega_c}{\pi}, & \text{if } n = \alpha \end{cases}$$

$$h_d(n) = \begin{cases} \dfrac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}, & \text{if } n \ne \alpha \\ \dfrac{\omega_c}{\pi}, & \text{if } n = \alpha \end{cases}$$

(B)High Pass Filter:

High pass filter allows only high frequency signals over the range $-\pi \le \omega \le -\omega_C$ and $\omega_C \le \omega \le \pi$ and attenuate all other low frequency signals. Frequency response of ideal and practical characteristics of a high pass filter as shown below.



$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha}, & -\pi \le \omega \le -\omega_c \text{ and } \omega_c \le \omega \le \pi \\ 0, & -\omega_c \le \omega \le \omega_c \end{cases}$$

Impulse response of a desired filter can be obtained from IDTFT

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left(\int_{-\pi}^{-\omega_c} e^{-j\omega\omega} e^{j\omega n}\, d\omega + \int_{\omega_c}^{\pi} e^{-j\omega\omega} e^{j\omega n}\, d\omega\right)$$

$$= \frac{1}{2\pi}\left(\int_{-\pi}^{-\omega_c} e^{j\omega(n-\alpha)}\, d\omega + \int_{\omega_c}^{\pi} e^{j\omega(n-\alpha)}\, d\omega\right)$$

$$= \frac{1}{2\pi}\left(\left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{-\pi}^{-\omega_c} + \left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{\omega_c}^{\pi}\right)$$

$$= \frac{1}{2\pi}\left(\frac{e^{-j\omega_c(n-\alpha)} - e^{-j\pi(n-\alpha)}}{j(n-\alpha)} + \frac{e^{j\pi(n-\alpha)} - e^{j\omega_c(n-\alpha)}}{j(n-\alpha)}\right)$$

$$= \frac{1}{2\pi}\left(\frac{e^{j\pi(n-\alpha)} - e^{-j\pi(n-\alpha)} - \left(e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}\right)}{j(n-\alpha)}\right)$$

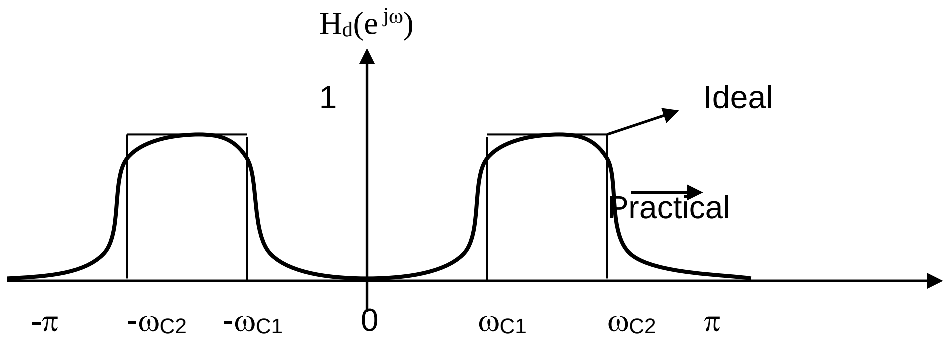
$$= \frac{1}{2\pi}\left(\frac{2j\sin[\pi(n-\alpha)] - 2j\sin[\omega_c(n-\alpha)]}{j(n-\alpha)}\right)$$

$$= \left(\frac{\sin[\pi(n-\alpha)] - \sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}\right)$$

$$= \begin{cases} \dfrac{\sin[\pi(n-\alpha)] - \sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}, & \text{if } n \ne \alpha \\ 1 - \dfrac{\omega_c}{\pi}, & \text{if } n = \alpha \end{cases}$$

(C)Band Pass Filter:

Band pass filter allows only a certain band of frequency signals over the range $-\omega_{C2} \le \omega \le -\omega_{C1}$ and $\omega_{C1} \le \omega \le \omega_{C2}$ and attenuate all other frequency signals. Frequency response of ideal and practical characteristics of a band pass filter as shown below.



$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha}, & -\omega_{c2} \le \omega \le -\omega_{c1} \text{ and } \omega_{c1} \le \omega \le \omega_{c2} \\ 0, & -\pi \le \omega \le -\omega_{c2} \ \& -\omega_{c1} \le \omega \le \omega_{c1} \ \& \ \omega_{c2} \le \omega \le \pi \end{cases}$$

Impulse response of a desired filter can be obtained from IDTFT

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left(\int_{-\omega_{c_2}}^{-\omega_{c_1}} e^{-j\omega\omega} e^{j\omega n}\, d\omega + \int_{\omega_{c_1}}^{\omega_{c_2}} e^{-j\omega\omega} e^{j\omega n}\, d\omega\right)$$

$$= \frac{1}{2\pi}\left(\int_{-\omega_{c_2}}^{-\omega_{c_1}} e^{j\omega(n-\alpha)}\, d\omega + \int_{\omega_{c_1}}^{\omega_{c_2}} e^{j\omega(n-\alpha)}\, d\omega\right)$$

$$= \frac{1}{2\pi}\left(\left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{-\omega_{c_2}}^{-\omega_{c_1}} + \left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{\omega_{c_1}}^{\omega_{c_2}}\right)$$

$$= \frac{1}{2\pi}\left(\frac{e^{-j\omega_{c_1}(n-\alpha)} - e^{-j\omega_{c_2}(n-\alpha)}}{j(n-\alpha)} + \frac{e^{j\omega_{c_2}(n-\alpha)} - e^{j\omega_{c_1}(n-\alpha)}}{j(n-\alpha)}\right)$$

$$= \frac{1}{2\pi}\left(\frac{e^{j\omega_{c_2}(n-\alpha)} - e^{-j\omega_{c_2}(n-\alpha)} - \left(e^{j\omega_{c_1}(n-\alpha)} - e^{-j\omega_{c_1}(n-\alpha)}\right)}{j(n-\alpha)}\right)$$

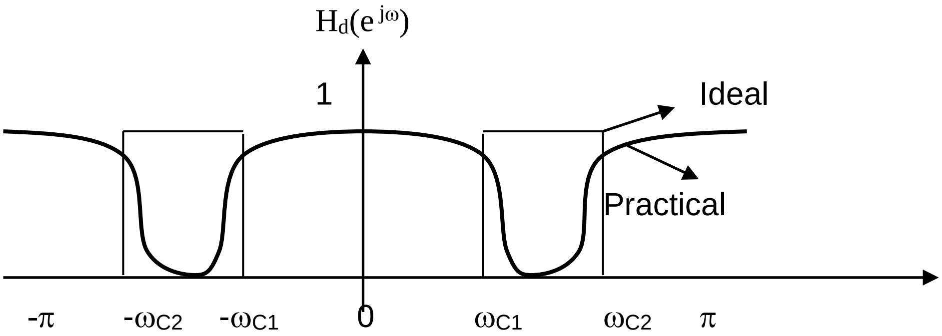
$$= \frac{1}{2\pi}\left(\frac{2j\sin[\omega_{c_2}(n-\alpha)] - 2j\sin\omega_{c_1}(n-\alpha)]}{j(n-\alpha)}\right)$$

$$= \left(\frac{\sin[\omega_{c_2}(n-\alpha)] - \sin[\omega_{c_1}(n-\alpha)]}{\pi(n-\alpha)}\right)$$

$$= \begin{cases} \dfrac{\sin[\omega_{c_2}(n-\alpha)] - \sin[\omega_{c_1}(n-\alpha)]]}{\pi(n-\alpha)}, & \text{if } n \ne \alpha \\ \dfrac{\omega_{c_2} - \omega_{c_1}}{\pi}, & \text{if } n = \alpha \end{cases}$$

(D)Band Stop Filter:

Band stop filter allows entire band of frequency signals over the range $-\pi \le \omega \le -\omega_{C2}$ & $-\omega_{C1} \le \omega \le \omega_{C1}$ & $\omega_{C2} \le \omega \le \pi$.except unwanted band of frequency signals. Frequency response of ideal and practical characteristics of a band stop filter as shown below.



$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha}, -\pi \le \omega \le -\omega_{c2} \; \& -\omega_{c1} \le \omega \le \omega_{c1} \; \& \; \omega_{c2} \le \omega \le \pi \\ 0, -\omega_{c2} \le \omega \le -\omega_{c1} \; and \; \omega_{c1} \le \omega \le \omega_{c2} \end{cases}$$

Impulse response of a desired filter can be obtained from IDTFT

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left(\int_{-\pi}^{-\omega_{c_2}} e^{-j\omega\omega} e^{j\omega n}\, d\omega + \int_{-\omega_{c_1}}^{\omega_{c_1}} e^{-j\omega\omega} e^{j\omega n}\, d\omega + \int_{\omega_{c_2}}^{\pi} e^{-j\omega\omega} e^{j\omega n}\, d\omega\right)$$

$$= \frac{1}{2\pi}\left(\int_{-\pi}^{-\omega_{c_2}} e^{j\omega(n-\alpha)}\, d\omega + \int_{-\omega_{c_1}}^{\omega_{c_1}} e^{j\omega(n-\alpha)}\, d\omega + \int_{\omega_{c_2}}^{\pi} e^{j\omega(n-\alpha)}\, d\omega\right)$$

$$= \frac{1}{2\pi}\left(\left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{-\pi}^{-\omega_{c_2}} + \left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{-\omega_{c_1}}^{\omega_{c_1}} + \left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{\omega_{c_2}}^{\pi}\right)$$

$$= \frac{1}{2\pi}\left(\frac{e^{-j\omega_{c_2}(n-\alpha)} - e^{-j\pi(n-\alpha)}}{j(n-\alpha)} + \frac{e^{j\omega_{c_1}(n-\alpha)} - e^{-j\omega_{c_1}(n-\alpha)}}{j(n-\alpha)} + \frac{e^{j\pi(n-\alpha)} - e^{j\omega_{c_2}(n-\alpha)}}{j(n-\alpha)}\right)$$

$$= \frac{1}{2\pi}\left(\frac{e^{j\pi(n-\alpha)} - e^{-j\pi(n-\alpha)} + e^{j\omega_{c_1}(n-\alpha)} - e^{-j\omega_{c_1}(n-\alpha)} - \left(e^{j\omega_{c_2}(n-\alpha)} - e^{-j\omega_{c_2}(n-\alpha)}\right)}{j(n-\alpha)}\right)$$

$$= \frac{1}{2\pi}\left(\frac{2j\sin[\pi(n-\alpha)] + 2j\sin[\omega_{c_1}(n-\alpha)] - 2j\sin[\omega_{c_2}(n-\alpha)]}{j(n-\alpha)}\right)$$

$$= \left(\frac{\sin[\pi(n-\alpha)] + \sin[\omega_{c_1}(n-\alpha)] - \sin[\omega_{c_2}(n-\alpha)]}{\pi(n-\alpha)}\right)$$

$$= \begin{cases} \dfrac{\sin[\pi(n-\alpha)] + \sin[\omega_{c_1}(n-\alpha)] - \sin[\omega_{c_2}(n-\alpha)]}{\pi(n-\alpha)}, & \text{if } n \ne \alpha \\ 1 - \dfrac{\omega_{c_2} - \omega_{c_1}}{\pi}, & \text{if } n = \alpha \end{cases}$$

Window Functions and its Characteristics:

Window functions are finite duration sequences, which are used in the design of FIR digital LPF/HPF/BPF/BSF, to obtain the finite number of samples of impulse response h(n) from the infinite number of samples of hd(n). Various types of window functions used in the FIR filter design are given below.

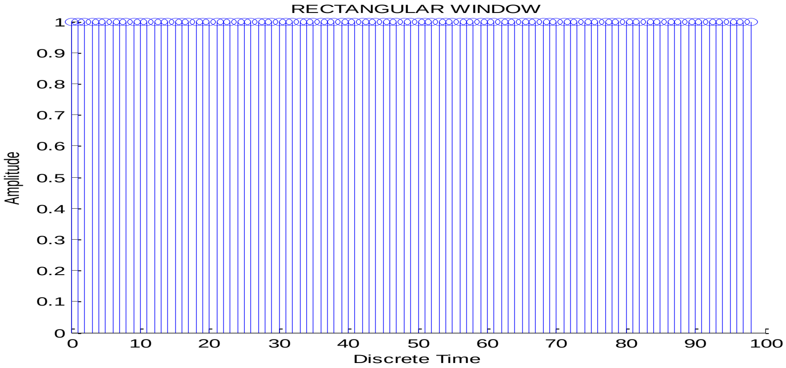
- Rectangular Window, $w_R(n)$
- Triangular/Bartlett Window, $w_T(n)$
- Hanning Window, $w_C(n)$ } Cosine Windows
- Hamming Window, $w_H(n)$ } Cosine Windows
- Blackman Window, $w_B(n)$ } Cosine Windows
- Kaiser Window, $w_K(n)$

(A)Rectangular Window, $w_R(n)$:

Rectangular window function in time domain over the range $0 \le n \le N-1$ can be defined as

$$w_R(n) = 1; \quad 0 \le n \le N-1$$

Example: if N=101, then $w_R(n) = 1; \; 0 \le n \le 100$.



Frequency domain characteristics of Rectangular window can be obtained by using DTFT

$$W_R(e^{j\omega}) = \sum_{n=-\infty}^{\infty} w_R(n)\, e^{-j\omega n}$$

$$= \sum_{n=0}^{N-1} (1)\, e^{-j\omega n}$$

$$= \sum_{n=0}^{N-1} \left(e^{-j\omega}\right)^n$$

$$= \frac{1-\left(e^{-j\omega}\right)^N}{1-e^{-j\omega}}$$

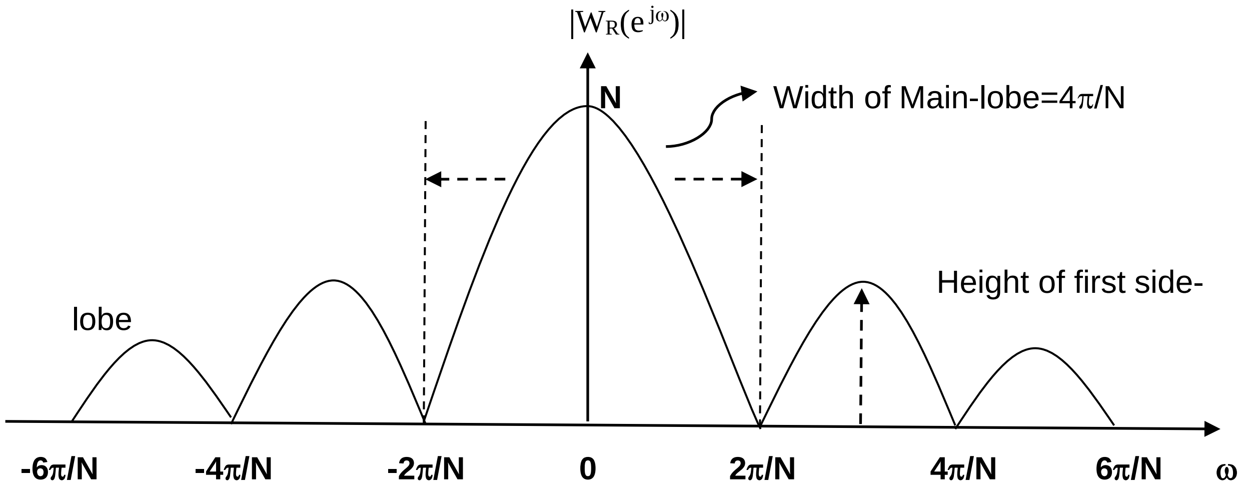
$$= \frac{1-e^{-j\omega N}}{1-e^{-j\omega}}$$

$$= \frac{1-e^{-j\omega N}}{1-e^{-j\omega}}$$

$$= \frac{e^{-j\omega N/2}\left(e^{j\omega N/2} - e^{-j\omega N/2}\right)}{e^{-j\omega/2}\left(e^{j\omega/2} - e^{-j\omega/2}\right)}$$

$$W_R(e^{j\omega}) = e^{-j\omega(N-1)/2}\frac{2j\sin(N\omega/2)}{2j\sin(\omega/2)}$$
$$= e^{-j\omega(N-1)/2}\frac{\sin(N\omega/2)}{\sin(\omega/2)}$$

$$\text{Magnitude, } \left|W_R(e^{j\omega})\right| = \left|\frac{\sin(N\omega/2)}{\sin(\omega/2)}\right| \ \& \ \textit{Phase}\angle W_R(e^{j\omega}) = -\omega\left(\frac{N-1}{2}\right)$$

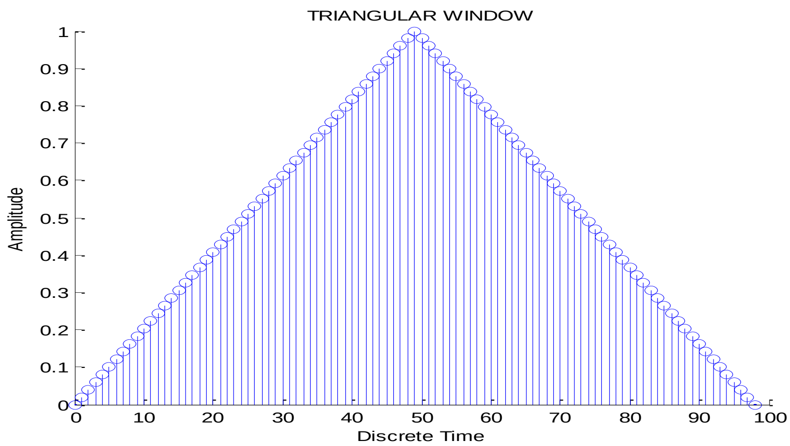


(B)Triangular or Bartlett Window, $w_T(n)$:

Triangular or Bartlett window function in time domain over the range $0 \le n \le N-1$ can be defined as

$$w_T(n) = 1 - \frac{2\left|n - \frac{N-1}{2}\right|}{N-1}\ ;\ 0 \le n \le N-1.$$

Example: if N=101, then $w_T(n) = 1 - \frac{|n-50|}{50};\ 0 \le n \le 100$



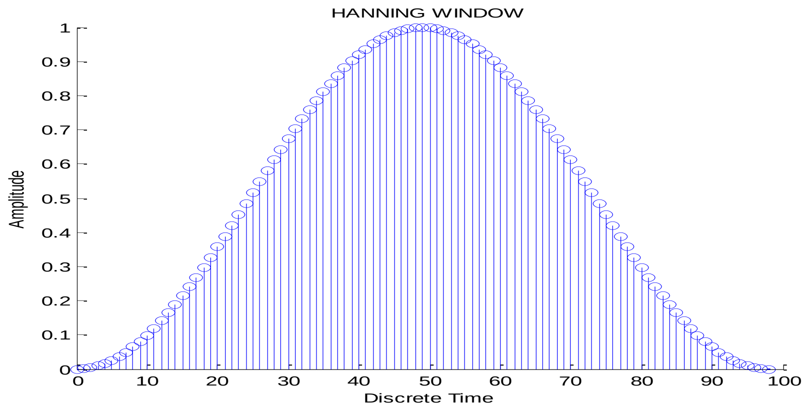
Width of Main-lobe=8π/N

(C)Hanning or Hann Window, $w_C(n)$: It comes under cosine window

Hanning or Hann window function in time domain over the range $0 \le n \le N-1$ can be defined as

$$w_c(n) = 0.5 - 0.5\,\text{Cos}\left(\frac{2n\pi}{N-1}\right);\ 0 \le n \le N-1.$$

Example: if N=101, then $w_C(n) = 0.5 - 0.5\,\text{Cos}\left(\frac{n\pi}{50}\right);\ 0 \le n \le 100$



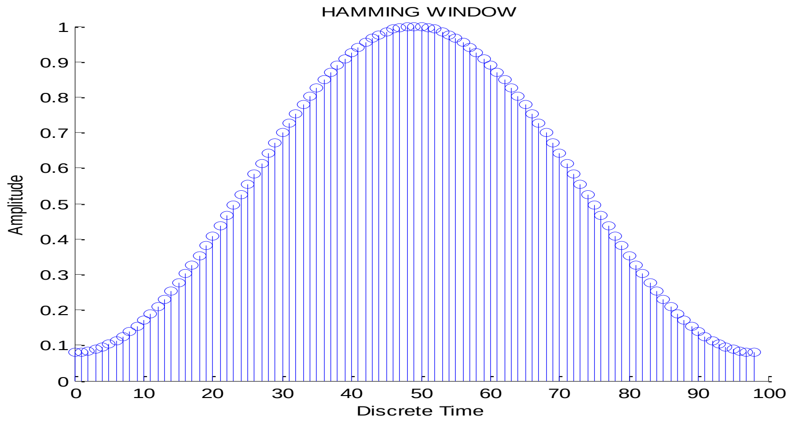
Width of Main-lobe=8π/N

(D)Hamming or Hamm Window, $w_H(n)$: It comes under cosine window

Hamming or Hamm window function in time domain over the range $0 \le n \le N-1$ can be defined as

$$w_H(n) = 0.54 - 0.46\,\text{Cos}\left(\frac{2n\pi}{N-1}\right);\ 0 \le n \le N-1.$$

Example: if N=101, then $w_H(n) = 0.54 - 0.46\,\text{Cos}\left(\frac{n\pi}{50}\right);\ 0 \le n \le 100$



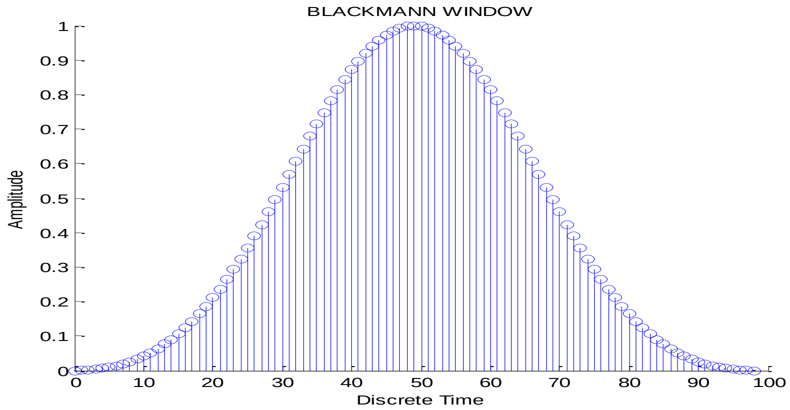
Width of Main-lobe=8π/N

(E)Blackman Window, $w_B(n)$: It comes under cosine window

Blackman window function in time domain over the range $0 \le n \le N-1$ can be defined as

$$w_B(n) = 0.42 - 0.5\,Cos\left(\frac{2n\pi}{N-1}\right) + 0.08\,Cos\left(\frac{4n\pi}{N-1}\right);\ 0 \le n \le N-1.$$

Example: if N=101, then $w_B(n) = 0.42 - 0.5\,Cos\left(\frac{n\pi}{50}\right) + 0.08\,Cos\left(\frac{n\pi}{25}\right);\ 0 \le n \le 100$

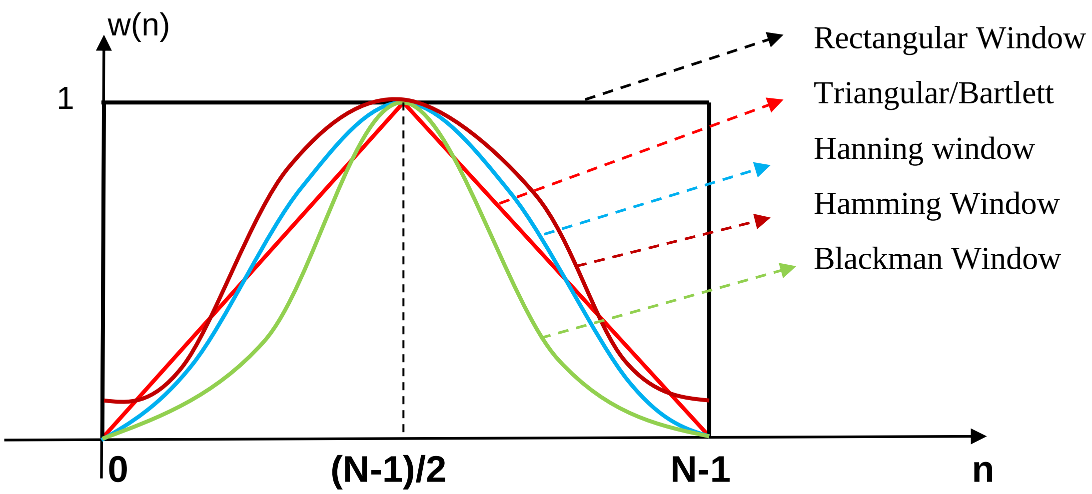


Width of Main-lobe=$12\pi/N$

(F)Frequency domain characteristics of Windows:

S.No.	Type of Window Function	Approximate width of main-lobe
1	Rectangular Window, $w_R(n)$	$4\pi/N$
2	Triangular/Bartlett Window, $w_T(n)$	$8\pi/N$
3	Hanning Window, $w_C(n)$	$8\pi/N$
4	Hamming Window, $w_H(n)$	$8\pi/N$
5	Blackman Window, $w_B(n)$	$12\pi/N$

(G)Time domain characteristics of Windows:



Steps to Design Digital LPF/HPF/BPF/BSF through FIR Methods:

Step 1:
Choose the frequency response of desired filter (LPF/HPF/BPF/BSF)

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha}, \text{over the desired band of frequencies} \\ 0 \quad , \text{over the unwanted band of frequencies} \end{cases}$$

Where, ω is digital frequency in rad/sec and α is constant

Step 2
Determine the impulse response of desired filter by using IDTFT

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

Step 3
In general, $h_d(n)$ contains infinite number of samples, but to design FIR filter consider only finite number of samples (N) by using Fourier series method or window technique.

(a) Fourier series method : $h(n) = h_d(n),\ 0 \le n \le N-1$.

(b) Window Technique : $h(n) = h_d(n)w(n)$, where $w(n)$ is window function, $0 \le n \le N-1$

Step 4:
Determine the transfer function of digital filter H(z) by using z-transform

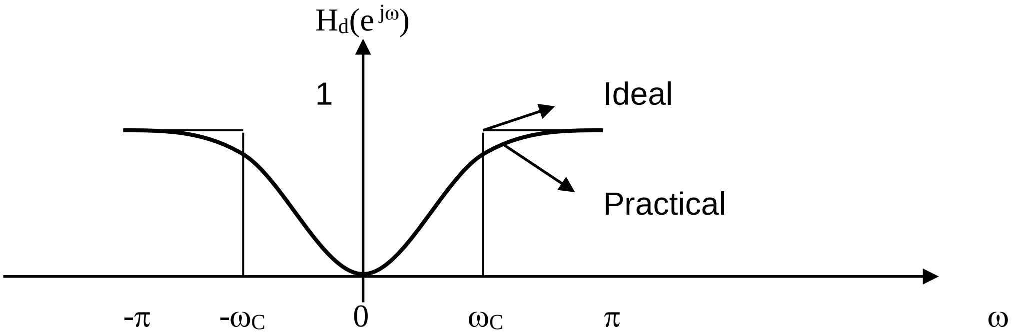
$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$

Step 5:
Finally design the linear phase FIR filter

Example: Design a digital high pass filter through FIR method by considering 7 samples of impulse response with a cutoff frequency of 0.8π rad/sample by using hamming window.

Step 1:
Choose the frequency response of desired filter (LPF/HPF/BPF/BSF)

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha},\ -\pi \le \omega \le -\omega_c \text{ and } \omega_c \le \omega \le \pi \\ 0, \qquad\qquad -\omega_c \le \omega \le \omega_c \end{cases}$$



Given,

Cutoff frequency $\omega_C = 0.8\pi$ rad/sample,

Number of samples N = 7,

Constant $\alpha = (N-1)/2 = 3$

Step 2:

Determine the impulse response of desired filter by using IDTFT

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left(\int_{-\pi}^{-\omega_c} e^{\text{-}j\omega\omega}\, e^{j\omega n}\, d\omega + \int_{\omega_c}^{\pi} e^{\text{-}j\omega\omega}\, e^{j\omega n}\, d\omega\right)$$

$$= \frac{1}{2\pi}\left(\int_{-\pi}^{-\omega_c} e^{j\omega(n-\alpha)}\, d\omega + \int_{\omega_c}^{\pi} e^{j\omega(n-\alpha)}\, d\omega\right)$$

$$= \frac{1}{2\pi}\left(\left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{-\pi}^{-\omega_c} + \left.\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right|_{\omega_c}^{\pi}\right)$$

$$= \frac{1}{2\pi}\left(\frac{e^{-j\omega_c(n-\alpha)} - e^{-j\pi(n-\alpha)}}{j(n-\alpha)} + \frac{e^{j\pi(n-\alpha)} - e^{j\omega_c(n-\alpha)}}{j(n-\alpha)}\right)$$

$$= \frac{1}{2\pi}\left(\frac{e^{j\pi(n-\alpha)} - e^{-j\pi(n-\alpha)} - \left(e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}\right)}{j(n-\alpha)}\right)$$

$$= \frac{1}{2\pi}\left(\frac{2j\sin[\pi(n-\alpha)] - 2j\sin[\omega_c(n-\alpha)]}{j(n-\alpha)}\right)$$

$$= \left(\frac{\sin[\pi(n-\alpha)] - \sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}\right)$$

$$= \begin{cases} \dfrac{\sin[\pi(n-\alpha)] - \sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}, & \text{if } n \neq \alpha \\ 1-\dfrac{\omega_c}{\pi}, & \text{if } n = \alpha \end{cases}$$

$$= \begin{cases} \dfrac{\sin[\pi(n-3)] - \sin[0.8\pi(n-3)]}{\pi(n-3)}, & \text{if } n \neq 3 \\ 1-\dfrac{0.8\pi}{\pi} = 0.2, & \text{if } n = 3 \end{cases}$$

Step 3:

In general, $h_d(n)$ contains infinite number of samples, but to design FIR filter consider only finite number of samples (N) by using window technique.

Window Technique : $h(n) = h_d(n)w(n)$,

where $w(n)=w_H(n)$ is hamming window function, $0 \le n \le N-1$, N=7

$$w_H(n) = 0.54 - 0.46\,\text{Cos}\left(\frac{2n\pi}{N-1}\right);\ 0 \le n \le N-1.$$

$$w_H(n) = 0.54 - 0.46\,\mathrm{Cos}\left(\frac{n\pi}{3}\right);\ 0 \le n \le 6.$$

Impulse Response of FIR Filter h(n)

$$h(n) = \begin{cases} \left(\dfrac{\sin[\pi(n-3)] - \sin[0.8\pi(n-3)]}{\pi(n-3)}\right)\left(0.54 - 0.46\,\mathrm{Cos}\left(\dfrac{n\pi}{3}\right)\right), & \text{if } n \ne 3 \\ \left(1 - \dfrac{0.8\pi}{\pi} = 0.2\right)\left(0.54 - 0.46\,\mathrm{Cos}\left(\dfrac{n\pi}{3}\right)\right) & \text{if } n = 3 \end{cases}$$

Compute 7 samples of h(n) by substituting n=0,1,2,3,4,5,6

$$n = 0 \Rightarrow h(0) = \left(\frac{\sin[\pi(0-3)] - \sin[0.8\pi(0-3)]}{\pi(0-3)}\right)\left(0.54 - 0.46\,\mathrm{Cos}\left(\frac{0\pi}{3}\right)\right)$$

$$= \left(\frac{-\sin[3\pi] + \sin[2.4\pi]}{-3\pi}\right)(0.54 - 0.46(1))$$

$$= \left(\frac{-0 + 0.9511}{-3\pi}\right)(0.54 - 0.46)$$

$$= -0.1009 \times 0.08$$

$$= -0.0081$$

$$n = 1 \Rightarrow h(1) = \left(\frac{\sin[\pi(1-3)] - \sin[0.8\pi(1-3)]}{\pi(1-3)}\right)\left(0.54 - 0.46\,\mathrm{Cos}\left(\frac{1\pi}{3}\right)\right)$$

$$= \left(\frac{-\sin[2\pi] + \sin[1.6\pi]}{-2\pi}\right)(0.54 - 0.46(0.5))$$

$$= \left(\frac{-0 - 0.9511}{-2\pi}\right)(0.54 - 0.23)$$

$$= 0.1514 \times 0.31$$

$$= 0.0469$$

$$n = 2 \Rightarrow h(2) = \left(\frac{\sin[\pi(2-3)] - \sin[0.8\pi(2-3)]}{\pi(2-3)}\right)\left(0.54 - 0.46\,\mathrm{Cos}\left(\frac{2\pi}{3}\right)\right)$$

$$= \left(\frac{-\sin[\pi] + \sin[0.8\pi]}{-\pi}\right)(0.54 - 0.46(-0.5))$$

$$= \left(\frac{-0 + 0.5878}{-\pi}\right)(0.54 + 0.23)$$

$$= -0.1871 \times 0.77$$

$$= -0.1441$$

$$n = 3 \Rightarrow h(3) = (0.2)\left(0.54 - 0.46\,\mathrm{Cos}\left(\frac{3\pi}{3}\right)\right)$$

$$= 0.2(0.54 - 0.46(-1))$$

$$= 0.2(0.54 + 0.46)$$

$$= 0.2 \times 1$$

$$= 0.2$$

$N = 4 \Rightarrow h(4) = h(2) = -0.1441$

$N = 5 \Rightarrow h(5) = h(1) = 0.0469$

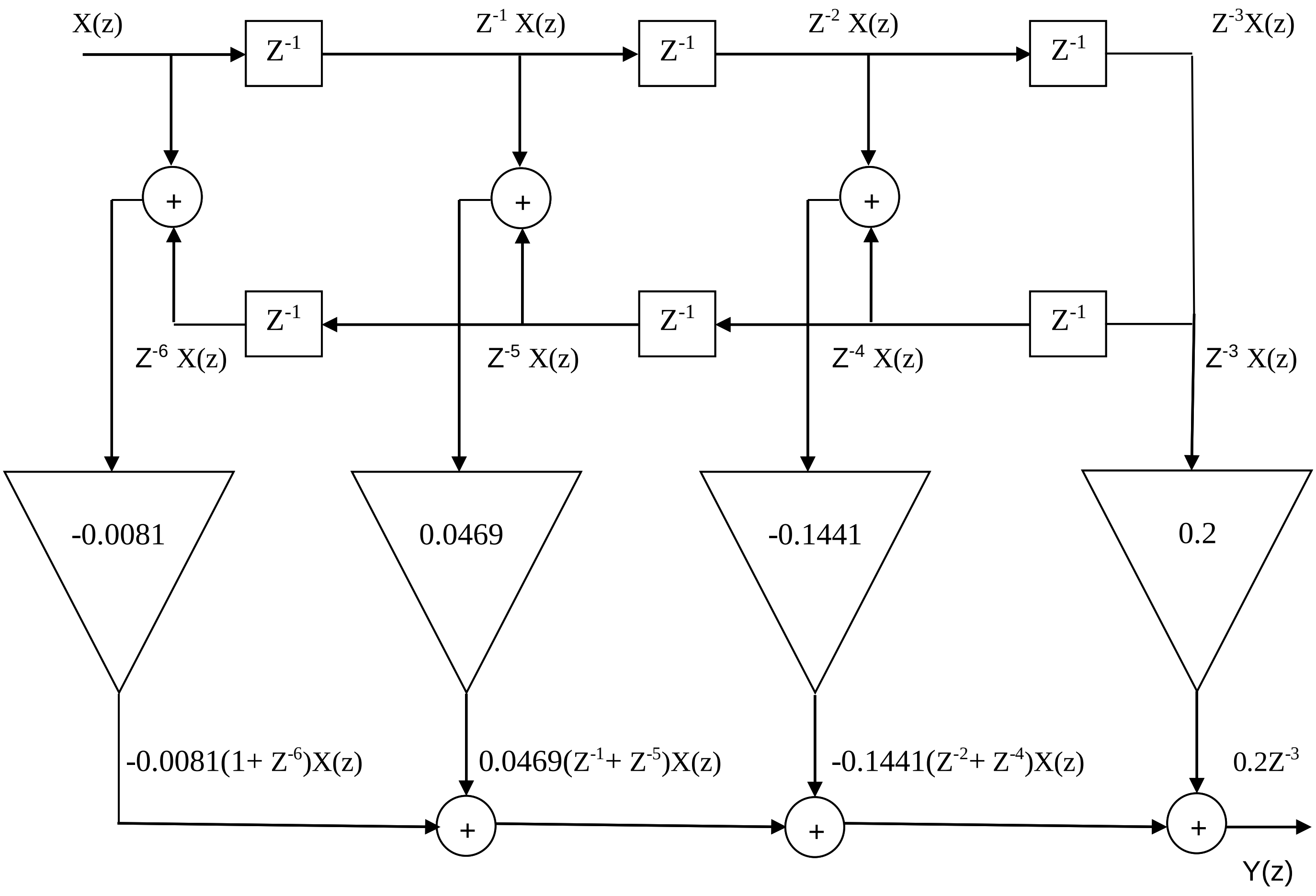
$N = 6 \Rightarrow h(6) = h(0) = -0.0081$

Impulse Response of FIR Filter h(n) = {-0.0081, 0.0469, -0.1441, 0.2, -0.1441, 0.0469, -0.0081}

Step 4:
Determine the transfer function of digital filter H(z) by using z-transform

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n}$$
$$= \sum_{n=0}^{6} h(n)\, z^{-n}$$
$$= h(0)\, z^{-0} + h(1)\, z^{-1} + h(2)\, z^{-2} + h(3)\, z^{-3} + h(4)\, z^{-4} + h(5)\, z^{-5} + h(6)\, z^{-6}$$
$$= -0.0081 + 0.0469 z^{-1} - 0.1441 z^{-2} + 0.2 z^{-3} - 0.1441 z^{-4} + 0.0469 z^{-5} - 0.0081 z^{-6}$$
$$= -0.0081(1 + z^{-6}) + 0.0469(z^{-1} + z^{-5}) - 0.1441(z^{-2} + z^{-4}) + 0.2 z^{-3}$$

Step 5:
Finally design the linear phase FIR filter

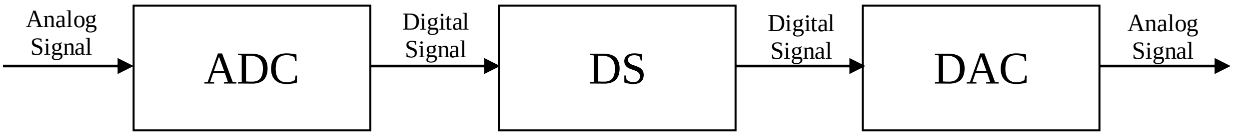


Comparison between FIR and IIR Filters:

S.No.	FIR Filters	IIR Filters
1	FIR filters are designed by considering only finite number of samples of impulse response	IIR filters are designed by considering all the infinite number of samples of impulse response
2	Linear phase FIR filters can be easily designed	Linear phase characteristics cannot be achieved
3	FIR digital filters can be directly designed to achieve the desired specifications	The design involves design of analog filter and then transforming analog filter to digital filter
4	Specifications include both magnitude and phase response	Specifications include only magnitude response
5	FIR digital filters are designed through Fourier series method or by using Window techniques	IIR digital filters are designed through approximation procedures and transformation techniques

Digital Signal Processing System:

Block diagram representation of Digital Signal Processing (DSP) system as shown below, where Analog to Digital Converter (ADC), Digital System (DS) and Digital to Analog Converter (DAC) are connected in cascaded.



The real-world (real time) signal is in analog form and it can be converted into digital signal by using ADC. It involves both sampling and quantization and the digital signal produced by ADC is processed by Digital System (DS). The Digital System (DS) is specially designed programmable hardware for DSP or an algorithm/software running on a general purpose digital system like Personal Computer (PC). At the receiving end, the original real time signal can be recovered by DAC.

(A)Advantages of DSP:

- In DSP, the digital systems can be cascaded without any loading problems.
- Digital hardware is compact, reliable, less expensive and programmable.
- The performance of DSP system can be easily upgraded or modified because DSP systems are programmable.
- The digital systems are less sensitive to tolerances of component values.
- These are easily transported because the digital signals can be processed off line.
- It has a better control of accuracy in digital systems compared to analog systems.
- Sophisticated signal processing algorithms can be implemented by DSP method.
- Digital signals are easily stored on magnetic media such as magnetic tape without loss of quality of reproduction of signal.

(B)Limitations of DSP:

- **Bandwidth:** The digital communications require a greater bandwidth than analogue to transmit the same information.
- **System Complexity:** In DSP, due to the use of devices such as DAC and ADC, the system complexity increases.

(C)Applications of DSP:

The digital processing of signals plays a vital role in almost every field of science and technology. Some of the applications are listed below.

Biomedical Signal Processing:

- ECG is used to predict heart diseases.
- EEG is used to study normal and abnormal behavior of the brain.
- EMG is used to study the condition of muscles.
- X-Ray images are used to predict the bone fractures.
- Ultrasonic scan images of kidney and gall bladder is used to predict stones.
- Ultrasonic scan images of foetus are used to predict abnormalities in a baby.
- MRI scan is used to study minute inner details of any part of the human body.

Speech Signal Processing:

- Speech compression and decompression to reduce memory requirement of storage systems.
- Speech compression and decompression for effective use of transmission channels.
- Speech recognition for voice operated systems and voice based security based systems.
- Speech recognition for conversion of voice to text.
- Speech synthesis for various voice based warnings or announcements.

Image Processing:

- Image compression and decompression to reduce memory requirement of storage systems.
- Image compression and decompression for effective use of transmission channels.
- Speech recognition for security systems.
- Filtering operations on image to extract features or hidden information.

Audio and Video Signal Processing:

- Audio and video compression for storage in DVDs
- Music synthesis and composing using music keyboards
- Analysis of audio signals will be useful to design systems for special effects in audio systems like stereo, woofer, karaoke, equalizer, attenuator, etc.

Communication:

- Analysis of signals received from RADAR is used to detect flying objects and their velocity.
- Analysis of modulated signals in both time and frequency domain
- Echo and noise cancellation in transmission channels
- Generation and detection of signals in telephones

Power Electronics:

- Analysis of switching currents and voltages in power devices will help to reduce losses.
- Output signal analysis of converters and inverters.

Geology:

- The seismic signals are used to determine the magnitude of earthquakes and volcanic eruptions.
- The seismic signals are also used to predict nuclear explosions.
- The seismic noises are also used to predict the moment of earth layers.

Astronomy:

- Analysis of light received from a star is used to determine the condition of the star.
- Analysis of images of various celestial bodies gives vital information about them.

DESCRIPTIVE QUESTIONS:

1. Design a digital low pass filter through FIR method by considering 11 samples of impulse response with a cutoff frequency of 1kHz and a sampling frequency of 4kHz by using Fourier series method.
2. Design a digital high pass filter through FIR method by considering 7 samples of impulse response with a cutoff frequency of 1.5kHz and a sampling frequency of 5kHz by using Fourier series method.
3. Design a digital band pass filter through FIR method by considering 7 samples of impulse response to pass frequencies in the range 1.5kHz to 3kHz and a sampling frequency of 8kHz by using Fourier series method.
4. Design a digital band stop filter through FIR method by considering 7 samples of impulse response to reject frequencies in the range 1.5kHz to 3kHz and a sampling frequency of 8kHz by using Fourier series method.
5. Design a digital low pass filter through FIR method by considering 7 samples of impulse response with a cutoff frequency of 0.2π rad/sample by using rectangular window.
6. Design a digital high pass filter through FIR method by considering 7 samples of impulse response with a cutoff frequency of 0.8π rad/sample by using hamming window.
7. Design a digital band pass filter through FIR method by considering 7 samples of impulse response to pass frequencies in the range 0.4π to 0.65π rad/sample by using hanning window.
8. Design a digital band stop filter through FIR method by considering 7 samples of impulse response to reject frequencies in the range 0.4π to 0.65π rad/sample by using Blackman window.
9. Design a digital band stop filter through FIR method by considering 9 samples of impulse response to reject frequencies in the range 0.4π to 0.65π rad/sample by using Bartlett window.
10. Design a digital band stop filter through FIR method by considering 7 samples of impulse response to reject frequencies in the range 0.4π to 0.65π rad/sample by using rectangular window.

QUIZ QUESTIONS:

1.	Window techniques are used in the design of (A) IIR filters (B) FIR filters (C) Both (D) None	B
2.	Which of the following are raised cosine windows (A) Hanning (B) Hamming (C) Blackman (D) Barlet	A,B,C
3.	Which of the following filter design requires both magnitude and phase response (A) IIR filters (B) FIR filters (C) Both (D) None	B
4.	Which of the following filter design involves both analog and digital filters (A) IIR filters (B) FIR filters (C) Both (D) None	A
5.	Due to linear phase design of FIR filters (A)Number of delays reduces (B)Number of Adders reduces (C)Number of constant multipliers reduces. (D)All	C
6.	Which of the following is hanning window function over the range 0 to N – 1 (A) $w(n) = 0.5 - 0.5\,Cos\left(\frac{2n\pi}{N-1}\right)$ (B) $w(n) = 0.54 - 0.46\,Cos\left(\frac{2n\pi}{N-1}\right)$ (C) $w(n) = 0.42 - 0.5\,Cos\left(\frac{2n\pi}{N-1}\right) + 0.08\,Cos\left(\frac{4n\pi}{N-1}\right)$ (D) None	A
7.	Which of the following is hamming window function over the range 0 to N – 1 (A) $w(n) = 0.5 - 0.5\,Cos\left(\frac{2n\pi}{N-1}\right)$ (B) $w(n) = 0.54 - 0.46\,Cos\left(\frac{2n\pi}{N-1}\right)$ (C) $w(n) = 0.42 - 0.5\,Cos\left(\frac{2n\pi}{N-1}\right) + 0.08\,Cos\left(\frac{4n\pi}{N-1}\right)$ (D) None	B
8.	Which of the following is Blackman window function over the range 0 to N–1 (A) $w(n) = 0.5 - 0.5\,Cos\left(\frac{2n\pi}{N-1}\right)$ (B) $w(n) = 0.54 - 0.46\,Cos\left(\frac{2n\pi}{N-1}\right)$ (C) $w(n) = 0.42 - 0.5\,Cos\left(\frac{2n\pi}{N-1}\right) + 0.08\,Cos\left(\frac{4n\pi}{N-1}\right)$ (D) None	C
9.	the following with respect to window and approximated width of main lobe (A) Hanning (B) Hamming (C) Blackman (D) Barlet (E) Rectangular (i) $4\pi/N$ (ii) $6\pi/N$ (iii) $8\pi/N$ (iv) $10\pi/N$ (v) $12\pi/N$	A,B,D-iii C-v E-i
10.	The width of the main lobe in window spectrum can be reduced by increasing the length of	Window sequence N